Oscillating directional domains in a ring semiconductor laser

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Abstract—We analyze transverse effects in a macroscopic class-A ring semiconductor laser based on a broad stripe gain medium. While the simplest modelling of such a device suggests the existence of stable domains emitting in opposite directions, the presence of alternate oscillations in the device prevents the stability of these domains. We describe the experimental observation of alternate oscillations and the associated transverse dynamics. A numerical analysis which drops many common assumptions shows that the presence of travelling domain walls is a sufficient driving force for the alternate oscillations.

Index Terms—Laser dynamics, Ring lasers, Semiconductor lasers.

I. INTRODUCTION

The study of semiconductor ring lasers has recently undergone considerable progress, in particular with the exploration and control of a variety of dynamical effects based on their capability to emit in one or the other direction. In fact the simplest possible modelling suggests pure directional bistability as the only possible situation in bidirectional semiconductor ring laser, and therefore the control of this bistability has very soon been envisioned as a possible mechanism for optical data storage or processing. Of course the experimental phenomena have soon overcome this simple case with in particular the appearance of directional oscillations which have received the name of alternate oscillations [1].

In this regime the device emits alternatively in one and the other direction, which results in antiphase oscillation between the power emitted in clockwise (CW) and counterclockwise (CCW) directions. This regime can be theoretically modelled in a class-B laser by including linear coupling between the emission directions in addition of their obvious competition for the same gain medium. The coupling can also be attributed to a population grating spontaneously formed in the medium, in which case the coupling is of course not linear. While the initial modelling was restricted to a rate equation approach (in a single longitudinal and single transverse mode limit), some effort has recently been devoted to theoretical [2] and experimental [3] studies of multimode situations in presence of coupling between the two emission directions, both leading to mode-locked regimes. In any of these cases, the additional complexity brought in by transverse effects is generally left aside. This is often a very valid theoretical assumption for the good reason that monolithic microscopic ring lasers are designed such that the waveguide operates in a fully single transverse mode regime. However, recent numerical work [4], [5] has addressed the issue of transverse effects in high aspect ratio ring lasers (although in a class-A approximation) with the aim of controlling the formation of spatial structures with the same properties as laser cavity solitons [6], [7], [8]. These studies (contrary to usual theoretical approaches to laser cavity solitons [9], [10]) rely on directional bistability to provide the coexisting states which enable the stability of localized structures. In such a scheme, a bright localized state in the clockwise emission direction would also appear in the counterclockwise emission in the form of a dark localized state. In a way which is somewhat complementary to the initial modelling of the alternate oscillations in which transverse effects were neglected, the modelling of laser localized states in ring devices was based on simpler modelling which could not produce anything else than directional bistability. However, as was many times demonstrated experimentally, linear or nonlinear coupling between the two directions of emission can easily spoil this bistability for large range of parameter values. In this contribution, we address experimentally and numerically the issue of transverse effects in a semiconductor ring laser in presence of alternate oscillations. We show that spatial segregation takes place, which is a required condition for the stability of localized directional emission domains. We show numerically that the existence of this domain wall can be sufficient to cause alternate oscillations even in a class-A laser.

II. DEVICES AND EXPERIMENTAL SETUP

The ring laser on which the experimental data are obtained is based on a GaAs broad stripe amplifier (BSA) [11] mounted on a C-mount and antireflection coated on both sides such that the specified reflectivity is $3 \times 10^{-4}$. The length of the active medium is 4 mm and its width is 200 $\mu$m.

The temperature of the mount of the active medium is actively stabilized (0.01 °C) which allows up to 4 A current operation. All the optics used is coated for the near infrared. The active medium is enclosed in a ring cavity built with high reflectivity mirrors. A 45% beam splitter is inserted in the cavity to couple part of the light out of the ring cavity. Each of the clockwise and counterclockwise beams coming out from the output coupler are split in two parts such as to measure simultaneously different spatial regions in both emission directions. The resulting four beams are then directed towards four equal fast photodetector (B, B1, A and A1) with

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lasing mode power outside the ring is around 0 - 22 mW. Both direction of emission have the same threshold current $I_{th} = 2.6$ A with a different slope depending on alignment condition. The laser threshold is higher in this setup compared to [3] essentially due to the output coupler which is a 45% beam splitter instead of a beam sampler (coupling 1%).

The right inset shows the lasing spectra of the ring laser when lasing emission takes place. It is centered around 979.4 nm and is the same for both CW and CCW emission direction. The narrow-bandwidth emission has a linewidth (FWHM) of 0.3 nm. Although the spectrum is not resolved by the optical spectrum analyst, the laser is operating on multiple longitudinal modes of the ring cavity. Due to the broad-area gain region, transverse effects also take place as we document in the following.

### III. Impact of Transverse Dynamics on the Spectral Properties

![Spectra Diagram](image)

Fig. 3. (Color online) (a) Power Spectrum in the bidirectional case as function of current with the diaphragm open, (b) Power spectra at $I = 2.6$ A (c) Power spectra at $I = 2.8$ (d) Power spectra at $I = 3.1$ A.

Fig. 3(a) shows the power spectrum as function of current with the diaphragm open. The power spectrum displays several ring cavity modes beatings up to 3 GHz, the bandwidth of the RF spectrum analyzer. Figures 3(b),(c) and (d) show cut along the power spectra for three different current ($2.6$ A, $2.8$ A and $3.1$ A). From these figures it is possible to see that not only beat notes at the cavity round trip appear in the spectra but also that even very close to threshold the beat notes are split and their base is broadened. We interpret this structure to be at least partially caused by transverse effects since the spectra are strongly altered when the iris is closed as we show in the following.

![Power Spectrum Diagram](image)

Fig. 4. (Color online) (a) Power Spectrum in the bidirectional case as function of current with the diaphragm closed (b) Power spectrum at $I = 2.6$ A (c) Power spectrum at $I = 2.8$ (d) Power Spectrum at $I = 3.1$ A.

![Emission Spectra Diagram](image)

Fig. 2 shows the spontaneous emission spectra of the ring below threshold for increasing current level. The spontaneous emission has a Full Width Half Maximum (FWHM) of 20 nm and it is centered around 979 nm. It has the same value for both direction of emission (CW and CCW).

The left inset shows the time-averaged LI curve taken for CW (blue curve) and CCW (red curve) direction. The narrow-bandwidth emission has a linewidth (FWHM) of 0.3 nm.
Fig. 4(a) shows the power spectrum as function of current with the diaphragm closed. Figures 4(b),(c),(d) are cuts through the power spectra at different current values. In this case, only the beat notes are observed until a sufficient current level is reached (about 2.9 A). Above this value, satellite peaks appear around the beat notes. We attribute this simplification of the resonance structure to the absence of transverse effects. Even if some spurious backreflection on optical elements inside the cavity or on the semiconductor device facets are probably present (and could explain the satellite peaks), the fact that the satellite peaks are visible only for sufficiently high current values could indicate that the coupling between the two emission directions is of nonlinear origin (ie it could in principle result from a spatial population grating, necessarily very small due to carrier diffusion, which could be continuously seeded in presence of a standing wave).

In either of the above cases (ie with or without transverse effects), the absence of a relaxation oscillation peak is obvious on the spectra, which confirms that the device operates in a class-A regime.

IV. CROSS CORRELATION MEASUREMENT

In order to disclose the transverse dynamics in the system we perform cross correlation measurements, comparing different spatial regions and different emission directions. All the measurement taken in what follow are with the diaphragm open.

Fig. 5(a) show the cross correlation values between two identical detectors whose fiber inputs are placed on the same side of the ring laser. Both detectors are therefore monitoring the same emission direction. One of the fiber inputs is placed such that it captures only part of the emitted beam. The fiber input of the other detector can be transversally shifted such that it monitors different parts of the emitted beam.

It is made by scanning the micrometer screw on a xy translation stage where the coupled fiber-detector is attached. The pump current value is set at certain current level, where it is possible to see the alternate oscillation, (as shown on Fig. 5(c)) between CW and CCW. We then scan the position of the fiber input along the vertical direction and we monitor in real-time the cross correlation between the two detectors signals ie between the power emitted in the same direction but in different spatial regions. Fig. 5(a) shows that, it is possible to pass from a situation where the two time traces are almost perfectly correlated (when the two detectors monitor the same spatial region) to a situation where the time traces are perfectly anti-correlated when they monitor opposite sides of the device. The curve is reproducible moving up and down the detector. From this measurement, we conclude that alternate oscillations between clockwise and counterclockwise directions are associated to antiphase dynamics in the transverse dimension.

To complete the previous measurement, we show on Fig. 5(b) the time lag corresponding to the maximum correlation as function of distance between the two detectors. When the two detectors monitor the same spatial region the time lag of maximal correlation is of course 0. Upon increasing the distance between the monitored areas the highest correlation value is found for larger and larger delay up to 1 ns when the detectors monitor the opposite edges of the device.

Fig. 5(c) show the time trace of alternate oscillations observed between CW and CCW direction. The lag between the two traces is due to the cavity output beam splitter which is not centered in the cavity. When this spurious lag is compensated for (which we do in the following measurements), the time traces are anticorrelated which indicates that a power drop in the CW direction actually coincides in time with a power increase in the CCW direction in the active region. One of the emission directions is favoured with respect to the other (as observed from the DC component which in this case is much stronger in the CCW direction). This is strongly influenced by minute misalignments and corresponds to one of the LI curves showing stronger emitted power in Fig. 2.

Fig. 6 shows for a typical current value (3.1 A) in the alternate oscillations regime the cross correlation between the possible detector combinations. Detectors A and A1 monitor the opposite edges of the device in the CCW emission CCW
direction and detectors B and B1 monitor the opposite edges of the device in the CW direction. Detectors A and B (resp. A1 and B1) monitor the same spatial region in opposite emission directions. The small lags (maximum correlation or anticorrelation not strictly in 0) is attributed to the fact that the beam paths to each detector are slightly different. From the anticorrelation between detectors A and B (resp. A1 and B1) one concludes that the oscillations observed are indeed alternate oscillations since emission takes place mostly in either one or the other direction. In addition to this rather expected results, the spatial anticorrelations at 0 lag (between detectors A and A1 resp. B and B1) shows that anticorrelated dynamics takes place along the spatial dimension as well. The corollary of these two measurements are the perfect correlations (at 0 lag) between detectors A and B1 (resp A1 and B). At this point, one can conclude that in the particular situation measured above (more complex situations also exist), alternate oscillations in the emission direction are coupled with alternate oscillations in the transverse dimension. We note that although the observed dynamics is very intricate, the very high (anti-)correlation values either in space or time indicate that the dynamics is mostly deterministic.

This situation, which is a bit intricate, is schematically represented on Fig. 7. The arrows represent the direction of the beam inside the device: at a particular instant the top part of the device emits mostly in the clockwise direction while the bottom part of the device emits mostly in the counterclockwise direction. This picture then gets reverted in time during alternate oscillations, the top part now emitting clockwise and the bottom part emitting counterclockwise.

![Fig. 7. (Color online) Schematic representation of the temporal dynamics CW and CCW inside the devices.](image)

Although the situation illustrated above is robust, many other and more complex regimes can be observed in this experimental system which possesses many degrees of freedom. As an illustration of this complexity, we show on figure 8 the value of the cross-correlation coefficient (at 0 time lag) depending on the bias current for all detectors combination.

At low bias current no dynamics is visible in the time series, the emitting intensity appearing constant in time even when the power spectrum analyzer (due to much higher dynamical range than the oscilloscope) displays several beat notes. Therefore the cross-correlation signal between different areas and different direction also vanishes at low bias current. Cross-correlations are then observed only when the dynamics start to be visible on the time traces (at about 2.9 A). Although the cross correlation coefficient (at 0 time lag) between different areas vanishes for some bias values, we stress that different spatial regions are always strongly correlated since the cross-correlation never identically vanishes for non-zero lag.

V. THEORY

A. Introduction

As stated in the introduction, one of the motivations of the present study lies in a series of papers that have theoretically speculated about the possibility to used pair of interacting domain walls as cavity soliton [4], [5], [12], [13]. Because the envelope equations used in these studies were derived under rotating wave, plane wave, longitudinal single mode approximations and adiabatic first order reduction of the dynamics to those of the electric field envelopes (a situation in principle not very close to our experimental configuration), the paper [14] was initially devoted to a more detailed investigation of the domain walls separating counter-propagating traveling waves. Especially, have some physical mechanisms, potentially relevant, been washed out by the various approximations? In the following we discuss a potential interpretation of our measurement in terms of the mechanism reported in [14].

B. Choice of the level of description

On the basis of the previous experimental observations, we expect a configuration as those schematically displayed in fig.9. The transverse section is divided into two areas by a domain wall, which is extended along the propagation direction. On each side, a traveling wave propagating along the z optical axis takes place, but with an opposite direction. Possibly, the direction of the traveling wave may alternate with time. Of course the exact geometry of the domain wall is unknown: we expect a 2D surface, mainly along the z propagation axis and along y because of the aspect ratio of the transverse section. A priori, the domain wall width depends on the pump rate, on diffusion and refraction as well as the angle between the traveling wave vector and the normal to...
the domain wall surface. As in hydrodynamics, instabilities analogous to the famous Kelvin-Helmholtz instability of shear flows may develop and destroy the domain wall flatness.

Analogy with the hydrodynamics [15] and liquid crystal [16] situations is enlightening. In those systems the observations are straightforward and the fast space and time periodicities are visible by an human eye. Domains walls separating counter-propagating traveling waves are ubiquitous. Boundary conditions have been recognized to be involved in the large scale spatial distribution of the wave-vectors, such that the usual projection of the mean flow [17] onto the cavity modes is in general helpful. However far from the pattern threshold, the typical domain wall width is only about a few wavelengths such that its description would involve a huge number of cavity modes. Furthermore, the impossibility to accurately describe the short scale structures appearing in these systems in an envelope equation formalism has been noted, and this observation has motivated numerous theoretical investigations [17], [18], [19].

In non linear optics, domains walls have been investigated for waves propagating in the transverse plane, both analytically [20] and experimentally [21]. We develop on this subject because it is a perfect example of the difficulties to deal with multi-scale analysis and a good illustration of the paradox: short scales description may be required to identify the parameter range where slowly varying envelope descriptions are available. Assume an electromagnetic wave propagating along the z axis \( E = F(X,Y,T) e^{i(ωt−kz)} \), where the envelop \( F \) is expected to vary slowly in space and time with respect to the optical wave vector \( k \) and pulsation \( ω \). \( X \) and \( Y \) are the transverse coordinates. Because of diffraction or detuning, transverse traveling waves may occurs. Writing \( F(X,Y,T) = A(X,Y,T) e^{i(ΩT−κX)} + B(X,Y,T) e^{i(ΩT+κX)} \) with \( Ω << ω \) and \( κ << k \), one can derive a envelope description of these transverses waves, where \( A \) and \( B \) slowly vary with respect to \( Ω \) and \( κ \). Transverse domain walls are then topological defects which make the connection between a region of the transverse section where \( A \neq 0 \) and \( B = 0 \) to a region where \( A = 0 \) and \( B \neq 0 \). This previous description is very helpful, but also somewhat rough: numerous physical mechanisms are missing such as the exact position and shape of the domain wall or the periodic formation and annihilation of dislocations. Hence potential short wavelength instabilities of the flat domain wall can not be described and may therefore remain hidden. Warned of these restrictions, optical transverse domains walls have been investigated on the basis not of envelope description but of the numerical simulation of a 2D \((x,y)\) complex Swift-Hohenberg equation. With this mathematical model, which is believed to describe the pattern formation transverse instability of a continuous plane wave [22], [23] (i.e. "our \( F(X,Y,T) \) level"), it has then been predicted [20] that a flat domain wall \((x = cst e)\), separating a right tilted traveling wave \( e^{i(ωt−kz+gy)} \) from a left tilted one \( e^{i(ωt−kz+gy)} \), should be stable for a given range of parameters.

In the present case however, we are dealing with domain wall separating longitudinal traveling waves. Two modelling approaches (each with potential inaccuracies) can be followed: on the one hand, the description of the configuration displayed in fig.9 in terms of forward and backward electric field envelopes could be applied. This standard approach is reasonably easy to deal with numerically (even when using close to realistic parameter values and active medium description) and it allows to investigate the stability of the flat domain wall with respect to long space and time scale perturbations. However, it can by construction not describe eventual instabilities of the wall against short space and time scales perturbation. On the other hand, a detailed short scales description can describe such instabilities, but at a huge numerical cost. Such a description is indeed too time-consuming and beyond the reach of the today’s computers, even massively parallel ones. Therefore the numerical investigations have to be speeded up through an unphysical increase of the characteristic time scales and a simplistic material description. In spite of these obviously questionable points, we chose to analyze the system in this latter framework because the wall separating domains of longitudinal waves propagating in opposite direction is the exact feature we want to analyze in detail.

C. Mathematical model and boundary conditions

Although all the optics used in the experiment are anti-reflection coated, a small amount of backscattering still exists. We will not consider such a linear coupling here because we want to numerically demonstrate that, even in the absence of backscattering, the presence of a domain wall alone already leads to a periodic alternation of the traveling waves.

The geometry of the numerical simulation is shown in fig.9. The nonlinear active medium is in grey and typical aspect ratios are \( l_x \simeq 30λ_c \), \( l_y \simeq λ_c \), and \( l_z = 70λ_c \) where \( λ_c \) is the threshold optical wavelength. It is surrounded first by a free propagation zone and then by a technical area where perfect matching layer boundary conditions [24], [25] have been investigated on the basis of envelope description but of the numerical simulation of a 2D \((x,y)\) complex Swift-Hohenberg equation. With this mathematical model, which is believed to describe the pattern formation transverse instability of a continuous plane wave [22], [23] (i.e. "our \( F(X,Y,T) \) level"), it has then been predicted [20] that a flat domain wall \((x = cst e)\), separating a right tilted traveling wave \( e^{i(ωt−kz+gy)} \) from a left tilted one \( e^{i(ωt−kz+gy)} \), should be stable for a given range of parameters.

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been implemented. The two latter zones are required in order to simulate an infinite reflectionless medium.

The well-known Yee’s scheme [26] is used for the spatial discretization of the space derivatives. This scheme is known to conserve the energy and to be consistent with the divergence free constraints. Also the boundary conditions at the interface between the active dielectric medium and the free propagation zone are automatically satisfied.

Inside the dielectric nonlinear active medium, the Maxwell equations are expressed as

\[ \nabla \cdot \mathcal{D} = 0 \quad \nabla \times \mathcal{B} = 0 \]  
(1)

\[ \nabla \times \mathcal{E} = -\frac{\partial \mathcal{B}}{\partial t} - \sigma \mathcal{B} \quad c^2 \nabla \times \mathcal{H} = \frac{\partial \mathcal{D}}{\partial t} + \sigma \mathcal{D} \]  
(2)

where \( c \) is the light speed, \( \sigma \) is a phenomenological coefficient (breaking of the \( t \rightarrow -t \) symmetry) which stands for homogeneous losses and \( \mathcal{B} \) is the magnetic field. The displacement vector \( \mathcal{D} \) is related to the electric field \( \mathcal{E} \) through

\[ \mathcal{D} = \mathcal{E} + \frac{1}{\varepsilon_0} \left( \mathcal{P}_a + \mathcal{P}_q \right) \]  
(3)

where \( \mathcal{P}_a = \varepsilon_0 \lambda \mathcal{E} \) stands for the background polarization and \( \mathcal{P}_q = \mathcal{P}_a \) is the atomic polarization whose dynamics satisfy

\[
\begin{cases}
\frac{\partial^2 \mathcal{P}_a}{\partial t^2} = -\nu \frac{\partial \mathcal{P}_a}{\partial t} - \omega_a^2 \mathcal{P}_a - \frac{2a}{\hbar} N \mathcal{M} \mathcal{E} \\
\frac{\partial N}{\partial t} = \gamma (N_p - N) + \frac{2}{\hbar \omega_a} \mathcal{E} \frac{\partial \mathcal{P}_a}{\partial t} + d \nabla^2 N 
\end{cases}
\]  
(4)

where \( N \) stands for the population inversion, \( \nu^{-1} \) is the decay time for the polarization, \( \omega_a \) is the atomic pulsation of the active medium, \( \gamma^{-1} \) the decay time of the population inversion, \( d \) is the diffusion coefficient of the population inversion and \( N_p \) is the pump parameter. \( \mathcal{M} \) is the anisotrop dipole moment matrix

\[ \mathcal{M} = \begin{pmatrix} \mu_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]  
(5)

In the free propagation area, the electromagnetic field still satisfies eq.(1,2), but now \( \mathcal{E} = \mathcal{D} \). For technical reasons, we have used the same damping coefficient \( \sigma \) for the active and free propagation medium.

D. Results

Eq.(1-5) have been simulated on a 24 processors Mac Pro computer. Although in our numerical simulation, we have not used the genuine values for the various damping rates, but the higher ones \( \sigma \approx 5 \times 10^4 \text{Hz}, \nu \approx 10^4 \text{Hz} \) and \( \gamma \approx 5 \times 10^4 \text{Hz} \), typical runs extending over \( \approx 50 \) the slowest characteristic time \( \sigma^{-1} \), already require about one month of computation time. Therefore our numerical investigation is far from being exhaustive

We first checked that our model does belong to class A laser: i) when the laser is switched on, the output power monotonously increases with time and does not display any characteristic overshooting oscillation ii) in the absence of transverse pattern, numerical simulation as well as analytical computations show that continuous traveling waves are always stable solution. Then, starting from the previously obtained continuous traveling wave solution, we now perform the following transformation: For the half of the numerical box volume corresponding to \( x > 0 \), the solution \( (\mathcal{D}, \mathcal{B}, \mathcal{P}, N) \) is duplicated without any modification. For the other half \( x < 0 \), a \( \pi \) rotation around the \( y \) axis is applied. Then we let the initial fields so built evolve with time. After \( \approx 10 \sigma^{-1} \) a steady state regime is reached, but we let it evolve up to 50 characteristic times without notice any additional change. The solution obtained is steady but not stationary. At a given time a typical plot of \( < S_z > \approx \frac{\omega_a}{2\pi} \) looks like those displayed in fig.10. But with longer time, we do observe a periodic alternating regime where the forward wave transforms into a backward one, and vice versa (fig.11). Here the frequency of the alternation is \( \approx 50 \text{Mgh} \). However the comparison with the experimental observation is not straightforward because this frequency certainly depends on \( \sigma, \nu \) and \( \gamma \) and that we have not used their genuine values, but orders of magnitude higher ones.

The population inversion equation (4) is a diffusion equation where the electromagnetic contribution \( \mathcal{E} \cdot \frac{\partial \mathcal{P}_a}{\partial t} \) acts as a source term. Therefore in presence of steady standing waves, for example due to an external forcing, the population will display a spatial grating with a contrast inversely proportional to the diffusion coefficient.

Now in absence of external forcing a positive feedback loop can take place spontaneously provided the diffusion coefficient is low enough: presence of standing waves gives rise to a spatial grating, and spatial grating promotes the occurrence of standing waves. Semiconductors, which are characterized by a high diffusion coefficient, are a priori not concerned with these spatial grating instability.

Fig.12 shows the population inversion versus \( x \) and \( z \) respectively in absence (fig.12.a) and in presence (fig.12.b) of a domain wall. Obviously the spatial grating exists only in presence of the domain wall. The laser belongs to class A and the population grating is naturally so tiny that the...
E. Discussion

The alpha factor has a critical influence on both the transverse and the longitudinal multimode dynamics. It is known to be one of the main differences between two levels systems (our theoretical model) and semiconductors (our experimental set up). Due to the strong amplitude variations which are present at the domain wall, the phase-amplitude coupling could be at the origin of additional instability of the flat domain wall. More precisely, comparison with out of equilibrium chemical oscillating reaction, which also feature phase-amplitude coupling, suggests a stabilizing effect for a given sign of the alpha factor, and a destabilizing effect for the other sign.

Another point where the alpha factor could play an important role is in the spontaneous formation of the oscillating directional domains. In our simulation, the continuous homogeneous plane wave solution is always stable, in disagreement with the experimental observations. We observe steady oscillating directional domains only because of the special choice of our initial conditions. Introducing of the alpha factor could help to spontaneously destabilize the homogeneous traveling wave.

VI. SUMMARY AND CONCLUSIONS

We have presented an exhaustive study of transverse effects in a semiconductor ring laser based on a broad stripe active medium. Although the experimental device operates in a classical regime (as shown by the absence of relaxation oscillations and as can be inferred from the physical parameters of the ring cavity), we observe alternate oscillations between clockwise and counterclockwise emission directions. When transverse degrees of freedom are not constrained, this dynamics is associated to antiphase oscillations in the power emitted at each edge of the active medium. Although these observations do not confirm the existence of stationary localized directional emission as predicted in earlier numerical studies, the dynamics we observe do indicate that spatial segregation of the different emission directions take place. This spatial segregation implies the presence of a wall separating the two domains. It is important to remark that this domain wall is the basic ingredient required for the stability of localized structures in a ring laser configuration. We have analyzed numerically the nature of this domain wall and its impact on the emergence of alternate oscillations.

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